

The Nucleon as a Holographic Cheshire Cat ¹

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Abstract

The Cheshire cat principle emerges naturally from the holographic approach of the nucleon in terms of a bulk instanton. The cat hides in the holographic direction. I briefly review the one-nucleon problem in the holographic limit.

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1 In memorium

This paper is dedicated to the memory of Gerry Brown who has been my mentor and colleague for the past three decades. One day in the fall of 1982, my advisor Michel Baranger informed me that he has invited Gerry Brown to give a talk at MIT and meet me. He indicated briefly that Gerry was an old friend of his, that competed historically with him in the early days of the Lamb-shift calculation. (Later I learned from Gerry that Baranger beat him on the 10 Mega-cycle correction to the Feynman and Schwinger Lamb-shift landmark calculation. However, Gerry was always very proud of his higher-order corrections to the Lamb-shift in Coulombic atoms. He would always go in great details and pride about how he carried this important and difficult calculation, and also the physical effort it took him shuttling back-and-forth in Birmingham between the computing facility and his office.)

I met Gerry in the fall of 1982, the morning after a tumultuous talk at the Laboratory of Nuclear Science at MIT where Gerry was talking about the excited states of the little bag model. Needless to say that little bags were not popular in the large bag sanctum that afternoon. The meeting with Gerry took place in Feshbach office with Baranger and later Kerman present. I had a very good time discussing my several research projects with Gerry on the board. We departed at lunch and Gerry wished me well in my work.

Just after the new year in 1983 and out of the blue, I received a letter from Gerry offering me a 3-year position in Stony Brook. I was stunned since I have not applied to any place, let alone thought about graduating after just 3 years at MIT. After consulting with Baranger, I wrote back to Gerry accepting his offer. Upon graduation in the Spring of 1983, I was also offered an NSF NATO fellowship at the Niels Bohr Institute. Gerry immediately indicated that I should take it and his offer would still be valid at the end of the fellowship. He also mentioned that he will be in Copenhagen in the fall of 1983 as part of his dual appointment at the Nordic Institute.

In the summer of 1983 I flew from Boston to Copenhagen a country I never visited before. I arrived at the Niels Bohr at around noon time. Upon checking at the secretarial office, I was informed that Gerry has asked that I wait for him to be picked up. Soon after the secretary entered the sounding Morse-like call code on the famed inter-phone at the Niels Bohr, Gerry showed up to welcome me in this new place. He helped me carry my suitcase across the hall-ways of the institute introducing me to luminaries of the place. We only stopped by Mottelson office for an official introduction as I was going to be their NSF fellow for the next year. After leaving my suitcase in Gerry office we headed to the cafeteria where two tables were full of Gerry students, postdocs and collaborators. I was stunned by the

number of people around him.

After lunch Gerry asked me to follow him to his office. There we started chatting about some physics, while in typical Gerry style he pulled out two papers from his brief-case and handed them to me. He said that on his way to Copenhagen he passed by Princeton and there he met Witten and Nappi who told him about their new work on the Skyrme model. The two papers were the by now famed work by Witten and also Adkins, Nappi and Witten on the Skyrmion. Gerry asked me to explain them to him. While, I was about to be briefed on the logistics of the place Jim Lattimer and Gerry Cooperstein came in. I told Gerry that I would be able to figure out the place and left.

The following many weeks I had the pleasure to discuss the papers with Gerry and his students and collaborators. It was the beginning of a great journey by Gerry side. I owe much to Gerry in terms of mentoring, supporting and counseling during all my years with him. Gerry sense of physics and wit is unmatched. More importantly his humanity as measured by the amount of care and kindness he has shown to many of his students and collaborators is legend. For Gerry we were part of his family. For me he was one of the last eagles.

2 Introduction

Historically, quark bag models were simplified models of hadrons consisting of free quarks and gluons confined to a bag because of asymptotic freedom, and dressed up by mostly pions to account for the pion tail of baryons. The bag radius was initially considered measurable, with the current Jefferson facility being tasked to measure it. Two competing pictures emerged: the original MIT bag model with a large bag radius surrounded by a bare vacuum, and the Stony-Brook bag model with a small bag radius surrounded by pions [1].

It turns out that this delineation is unphysical at low energy, as demonstrated in the Cheshire cat principle [2]. Quantum effects and anomalies cause most of the charges (fermionic, axial, etc.) to leak making the bag boundary immaterial [3], much like the smile of the Cheshire cat in "Alice in wonderland" [4]. The Skyrme model is an example of this principle whereby the unphysical bag radius is reduced to zero size [5].

The Skyrme model has recently re-emerged from holographic QCD in the double limit of large N_c and strong coupling t'Hooft coupling $\lambda = g^2 N_c$ [6, 7]. The Skyrmon is the line-integral of a flavored instanton located along the holographic or 5th-dimension (see Eq. 5) below). Baryon number at the boundary is dual to instanton number in bulk [8]. Although the holographic correspondence is only proved for certain conformal gauge theories at the



Figure 1: Alice's Cheshire Cat.



Figure 2: The Cheshire Cat Principle.

boundary [9], it is usually assumed for non-conformal theories such as QCD. Many of the arguments presented below have been worked out in [10, 11].

3 The Skyrmion from the Instanton

In the large N_c limit baryons are chiral solitons. A first principle framework for discussing chiral solitons at large 't Hooft coupling $\lambda = g^2 N_c$ is the use of the probe $D8-\overline{D8}$ flavor branes in the $D4$ color brane set up as discussed by Sakai and Sugimoto [6]. After reduction, the result is an effective flavor Yang-Mills-Chern-Simons theory in a 5 dimensional curved background. More specifically, the leading contributions in a $1/\lambda$ expansion of the D-brane

Born-Infeld (DBI) effective action on D8 is [6],

$$\begin{aligned}
S &= S_{YM} + S_{CS} , \\
S_{YM} &= -\kappa \int d^4x dZ \operatorname{tr} \left[\frac{1}{2} K^{-1/3} \mathcal{F}_{\mu\nu}^2 + M_{KK}^2 K \mathcal{F}_{\mu Z}^2 \right] , \\
S_{CS} &= \frac{N_c}{24\pi^2} \int_{M^4 \times R} \omega_5^{U(N_f)}(\mathcal{A}) ,
\end{aligned} \tag{1}$$

where $\mu, \nu = 0, 1, 2, 3$ are 4D indices and the 5th coordinate is Z . Also $\kappa = \lambda N_c / 216\pi^3$. The confining scale in this gravity dual reduction is the Kaluza-Klein compactification of the $D4$ branes or M_{KK} . Throughout, all units will be expressed in terms of $M_{KK} = 1$ for convenience. The effect of the 5-dimensional gravity due to the $D4$ branes in bulk is the warping $K = 1 + Z^2$. \mathcal{A} is the 5D $U(N_f)$ 1-form gauge field and $\mathcal{F}_{\mu\nu}$ and $\mathcal{F}_{\mu Z}$ are the components of the 2-form field strength $\mathcal{F} = d\mathcal{A} - i\mathcal{A} \wedge \mathcal{A}$. $\omega_5^{U(N_f)}(\mathcal{A})$ is the Chern-Simons 5-form for the $U(N_f)$ gauge field

$$\omega_5^{U(N_f)}(\mathcal{A}) = \operatorname{tr} \left(\mathcal{A} \mathcal{F}^2 + \frac{i}{2} \mathcal{A}^3 \mathcal{F} - \frac{1}{10} \mathcal{A}^5 \right) , \tag{2}$$

In (1) baryons are flavor $SU(2)$ instantons of the reduced Yang-Mills-Chern-Simons theory. At large $\lambda = g^2 N_c$, the instanton is forced by the gravitational warping or K to localize near $Z \approx 0$ for otherwise the mass term in (1) is large. On the other hand, the topological Chern-Simons term is repulsive preventing the instanton to shrink to zero size. The balance of these two effects is an instanton of size $Z \approx 1/\sqrt{\lambda} \ll 1$ in units of M_{KK} [6].

For small size instantons localized near $Z \approx 0$, the effects of the gravitational warping on the instanton can be neglected to leading order. Thus the instanton $SU(2)$ flavor configuration \mathbb{A}_M and its supporting $U(1)$ Coulomb potential $\hat{\mathbb{A}}_M$ read

$$\hat{\mathbb{A}}_0 = -\frac{1}{8\pi^2 a \lambda} \frac{2\rho^2 + \xi^2}{(\rho^2 + \xi^2)^2} , \quad \mathbb{A}_M = \eta_{iMN} \frac{\sigma_i}{2} \frac{2x_N}{\xi^2 + \rho^2} , \tag{3}$$

with all other gauge components set to zero. Here $\xi^2 = \vec{x}^2 + Z^2$ and $\rho \sim 1/\sqrt{\lambda}$ is the instanton size. We refer to [6] for more details on the relevance of this configuration for baryons. The configuration has spherical symmetry and satisfies

$$(\mathbb{R}\mathbb{A})_Z = \mathbb{A}_Z(\mathbb{R}\vec{x}) , \quad (\mathbb{R}^{ab}\mathbb{A}^b)_i = \mathbb{R}_{ij}^T \mathbb{A}_j^a(\mathbb{R}\vec{x}) , \tag{4}$$

with $\mathbb{R}^{ab}\tau^b = \Lambda^+\tau^a\Lambda$ a rigid $\text{SO}(3)$ rotation, and Λ is its $\text{SU}(2)$ version. The classical baryon is the projected map or holonomy along the Z -direction

$$U^{\mathbb{R}}(x) = \Lambda \mathbf{P} \exp \left(-i \int_{-\infty}^{+\infty} dZ \mathbb{A}_Z \right) \Lambda^+ . \quad (5)$$

The corresponding Skyrmion is $U^{\mathbb{R}=1}(\vec{x}) = e^{i\vec{\tau}\cdot\vec{x}\mathbf{F}(\vec{x})}$ with the profile $\mathbf{F}(\vec{x}) = \pi|\vec{x}|/\sqrt{\vec{x}^2 + \rho^2}$.

The quantum baryon emerges from a semiclassical quantization of the classical source (3-5). To achieve this, we define

$$A_M(t, x, Z) = \mathbb{R}(t) [\mathbb{A}_M(x - X_0(t), Z - Z_0(t)) + C_M(t, x - X_0(t), Z - Z_0(t))] , \quad (6)$$

The collective coordinates \mathbb{R}, X_0, Z_0 and the fluctuations C in (6) form a redundant set. The redundancy is lifted by constraining the fluctuations to be orthogonal to the zero modes. This can be achieved either rigidly [12] or non-rigidly [13]. We choose the latter via the constraint $\int_{x=Z=0} d\xi C G^B \mathbb{A}_M$ with $(G^B)^{ab} = \epsilon^{aBb}$ the real generators of \mathbb{R} .

For the quasi-zero mode in Z , the non-rigid constraint is more natural to implement since this mode is mostly localized near $Z \approx 0$. It is also local and thus preserves causality. The vector fluctuations at the origin linearize through the usual modes

$$d^2\psi_n/dZ^2 = -\lambda_n\psi_n , \quad (7)$$

with $\psi_n(Z) \sim e^{-i\sqrt{\lambda_n}Z}$. In the spin-isospin 1 channel they are easily confused with the zero mode: $\partial_Z \mathbb{A}_i$ near the origin as we show in Fig. 3. Using the non-rigid constraint, we define a new set of modes

$$\psi'_n(Z) = \theta(|Z| - Z_C)\psi_n(Z) , \quad (8)$$

with $Z_C \sim \rho \sim 1/\sqrt{\lambda}$ which is the origin for large λ . The baryon at small $\xi < |Z_C|$ is described by a flat or uncurved instanton located at the origin (Cheshire cat). At large $\xi > |Z_C|$, the instanton sources the vector meson fields (cloud). This is a holographic realization of the Cheshire cat principle [2] whereby Z_C plays the role of the Cheshire cat smile. In a way Alice's Cheshire cat hides in the holographic direction.

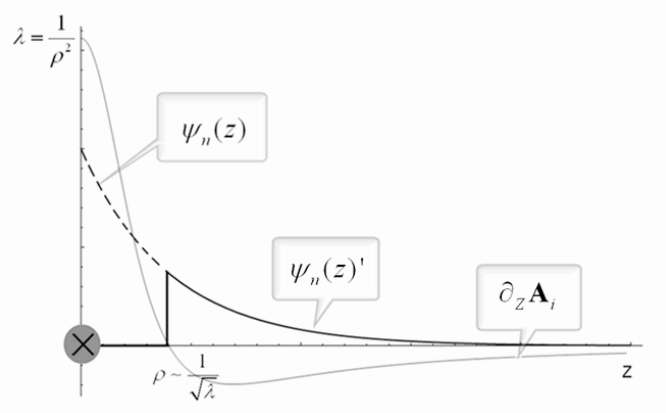


Figure 3: A vector fluctuation $\psi_n(Z)$ versus a quasi-zero mode $\partial_Z \mathbf{A}_i$. In the non-rigid gauge the new vector fluctuation is denoted by $\psi'_n(Z)$. See text.

4 The Baryonic Current

To extract the baryon current or nucleon form factor, we source the reduced action with $\hat{V}_\mu(x)$ a $U(1)_V$ flavor field on the boundary in the presence of the vector fluctuations or $C = \hat{v}$. The tree level baryonic current reads [10]

$$J_B^\mu(x) = -\kappa K \hat{\mathbb{F}}^{Z\mu}(x, Z) \left(1 - \sum_{n=1}^{\infty} \alpha_{v^n} \psi_{2n-1}\right) \Big|_{Z=B} - \sum_{n,m} m_{v^n}^2 a_{v^n} \psi_{2m-1} \int d^4y \kappa K \hat{\mathbb{F}}_{Z\nu}(y, Z) \Delta_{mn}^{\nu\mu}(y-x) \Big|_{Z=B} . \quad (9)$$

The massive vector meson propagator in Lorentz gauge is

$$\Delta_{\mu\nu}^{mn}(x) = \int \frac{d^4p}{(2\pi)^4} e^{-ipx} \left[\frac{-g_{\mu\nu} - p_\mu p_\nu / m_{v^n}^2}{p^2 + m_{v^n}^2} \delta^{mn} \right] , \quad (10)$$

The first contribution in (9) is the nucleon core instanton. The second contribution sums up the cloud of all vector mesons. The chief observation is that the core coupling vanishes due to the exact sum rule

$$\sum_{n=1}^{\infty} \alpha_{v^n} \psi_{2n-1} = 1 , \quad (11)$$

following from closure in curved space. *No cat is left*: vector Meson Dominance (VMD) is exact in holography. A similar argument holds for the pion electromagnetic form factor [6]. The results presented in this section were derived in [10]. They were independently arrived at in [14] using the strong coupling source quantization. They also support the bottom up effective approach described in [15] using the heavy nucleon expansion.

If we were to truncate the resonance contributions to the lowest vector mesons, the core contribution is non-zero. In this case, the deleniation of the Cheshire cat smile is no longer arbitrary. A specific position of the smile means a minimal number of vector mesons in the truncated cloud. For many years Gerry Brown has been advocating the 50/50 scenario for the baryon form factor using both phenomenology and his democratic principle.



Figure 4: Gerry Brown 50/50 scenario: Direct (left) + VMD (right). See text.

5 Acknowledgments

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